1. Introduction

In recent years, there has been an increasing interest in the socio-economic aspects of network systems. As an example, initiatives like the Euro-NF [2] and NSF FIND [3] promote economic incentives as a first-order concern in future network design. Economic models and pricing strategies for the edge of the networks enhance the profitability of local Internet Service Providers (ISPs) who sell Internet access for users.

Results in [4-6] initiate the discussion on customer loyalty and its impact on pricing strategies of ISPs, where a double reservation price based loyalty model is used. While [7] deals with ISPs' pricing strategies under uncertainties on static ISP markets, the authors of [8] used Stackelberg leader-follower game to handle dynamic local ISP market.

This paper presents and applies a realistic ISP customer loyalty model based on results of a survey carried out by our own. Under our price difference dependent loyalty model, the customers of the local ISPs are loyal based on the price difference of the prices of customers' current and possible future ISPs. Two price difference dependent loyalty models are used in this paper, a threshold-based model for analysing the effect of disloyal users on price setting strategies and a linear loyalty model for long-term strategies. The behaviour of service providers are investigated using game-theoretical tools.

There is broad literature in the area of modelling interactions between ISPs with game-theory, including [9-11]. They mostly assume a very simple user behaviour model: end-users choose the cheapest provider. However, this could be misleading if there are loyal customer segments present in the market, as loyalty is an important part of user behaviour. A vivid example of customer loyalty in practice is the loyalty contract between a service provider and a customer. The customers are charged with different price if they sign a contract and this difference depends on the length of the contract.

A number of empirical studies verify that user loyalty exists on ISP markets. In the USA 38% of the enterprise customers have been truly loyal to their ISPs [12]. National communication authorities of European Union’s countries carry out regularly market research dealing with customer loyalty towards local ISPs. In the UK 27% of broadband users have already switched their provider at least once [13], while in Ireland 84% of subscribers have not changed their ISP in the last 12 months [14]. In Portugal, 81% of broadband customers said they did not intend to change ISP in the following 12 months [15]. In addition, only 16% of the Finnish subscribers have switched their ISP in 2007, mainly because of a better offer from a competitor [16].

The paper is structured as follows. First, in Section 2 we make a case for a loyalty model which is based on the price difference of service providers. Section 3 shows the implications of a disloyal customer segment on ISPs' pricing strategies based on a game-theoretical analysis. Section 4 presents simulation results where we quantify the effect of uncertain a priori knowledge on market shares, prices, and profits. Section 5 concludes the paper.

2. A case for price difference dependent loyalty in ISP markets

In order to create a realistic user loyalty model toward ISPs we carried out by our own a survey to gain insights about customer loyalty towards ISPs. Based on almost 800 answers we state that loyalty exists in the Hungarian ISP market as more than half of the persons have...
not switched their ISP in the last five years. Numerous factors might have an impact on user loyalty, including Quality of Experience or customer service, but it turned out that the price difference of the ISPs also effects users' loyalty intentions toward service providers.

We asked what the minimal price difference is between the current and an other ISP when the answerers would switch their service providers. It was supposed that the two ISPs offer exactly the same service including connection speed, help center, etc. The answers are surprising as shown in Figure 1. Only around 5% of the answers would never leave their current ISPs, the remaining 95% said that there exists a price difference where they would become disloyal and would switch their providers. Based on the results we argue that modelling user loyalty based on the minimal price difference to switch is a realistic description of the ISP pricing problem.

3. Modeling loyalty based on price differences not only represents the relationship between the two prices but also includes information about the socio-economic aspects of the market. As an illustration consider two countries, a rich one and a poor one. In each country there exist two ISPs, they provide Internet access for 5 USD and for 10 USD. The price ratios are the same in both countries (0.5) but it is clear that there will be much more switchers in the poor country, where 5 USD (the price difference) worths a lot more than in the richer country.

The relation between the number of years to be a customer of the current ISP and the minimal price difference to switch is presented in Figure 2. The minimal price differences are not specific to the loyalty history of the subscribers, regardless of the years to be a subscriber of a specific ISP there are similar price differences where the customers would switch their ISPs.

3. Pricing Internet access for disloyal users

In this section we present our threshold-based loyalty model and apply it to analyse pricing strategies of service providers dealing with disloyal users. We investi-
provider to the market with users who have zero minimal price difference. The formal definition of game is:

• **Players:**
  the Internet Service Providers, \(i=1,\ldots,n\), \(ISPi\) has \(l_i\) loyal customers with \(d_i\) price difference.

• **Strategies:**
  the price of the Internet access, the decision of ISP \(i\) is \(p_i\), \(p_i \in [0, \alpha]\), players can have only pure strategies, they play single-shot game.

• **Payoff functions:**
  the payoff of ISP \(i\) is based on the above described payoff functions.

**Proposition**

If the ISPs' subscribers have different price differences there exists a pure strategy Nash equilibrium of the two-player game if the following conditions hold:

\[
\frac{l_2}{l_1 + l_2} \leq \frac{d_2}{\alpha}, \quad \frac{l_1}{l_1 + l_2} \leq \frac{d_1}{\alpha}
\]

**Proof**

We calculate the minimal prices where an ISP will compete in the \([0, \alpha-d]\) and \([\alpha-d, \alpha]\) intervals. We deal with ISP \(1\), we can have similar conditions for ISP \(2\):

\[
p_2 \in [0, \alpha-d], \quad p_2 > d_2 + \frac{l_1(d_1 + d_2)}{l_2}
\]

\[
p_2 \in [\alpha-d, \alpha], \quad p_2 > d_2 + \frac{l_1\alpha}{l_1 + l_2}
\]

This game has a best response figure where an intersection of the graphs does not exist except at \((\alpha, \alpha)\). ISP \(1\) will compete if her payoff can be larger if she grabs the users of ISP \(2\), namely

\[
(l_1 + l_2)(\alpha - d_2) > l_1\alpha
\]

\[
l_2\alpha > d(l_1 + l_2)
\]

\[
\frac{l_1}{l_1 + l_2} > \frac{d_2}{\alpha}
\]

The conclusion is the same for ISP \(2\). This means that if the ISPs' subscribers have different price sensitivities there exists a pure strategy Nash equilibrium at \((\alpha, \alpha)\) if the following conditions are satisfied:

The proposition can be generalised for \(N\) players, where the conditions of the Nash equilibrium are similar to the conditions of two player game.

With the above introduced model we are able to handle disloyal users by creating a virtual ISP which has the disloyal users. The price sensitivity of the subscribers of the virtual ISP is small (around zero). If we look at the constraints we can see that if we have disloyal users the game will no longer have pure strategy Nash equilibrium: the value of \(d_i/\alpha\) will be zero but the left side of the inequalities will be positive. As a closing word we state that if there are disloyal users in the market the Internet Service Providers can not play their pure equilibrium strategies, they have to set their prices based on probabilities and compete for the disloyal users.

### 4. Impact of long-term interaction on dynamic ISP markets

ISPs usually do not have complete knowledge about their competitors, they only have beliefs. They set their access prices based on their a priori information which will be adjusted based on the observed behaviour of the competitors. ISPs have to price Internet access not only in static markets, pricing strategies are even more crucial in dynamic markets if strategic decisions have to be made. We call a decision strategic if it has a significant long-term effect on the company. In this section, we analyse dynamic ISP markets, where a new ISP enters a local market using a linear price difference dependent loyalty model. We model game-theoretically the entry situation using a Stackelberg leader-follower game, where the incumbent ISPs are the leaders and the entrant ISP is the follower of the game.

The customers buy Internet access if the access price is at most \(\alpha\). The local ISP market consists of \(i=1,\ldots,n\) companies, ISP \(i\) has \(l_i\) loyal customers, thus the total number of the customers is \(\sum l_i\). If a subscriber changes her ISP then she selects the cheapest available price. The new, entering ISP has not got any subscribers at the beginning. We assume that the subscribers’ loyalty is based on the ISPs’ price difference, we use a linear customer loyalty function, meaning ISP \(i\) loses customers if ISP \(j\) has the lowest price (\(p_j < p_i\)). Every service provider plays rationally, namely selects its profit maximizing strategy. The discount factor is denoted by \(0 \leq \Theta \leq 1\), the profit of ISPs is discounted at each step with \(\Theta\). We assume that each ISP has the same discount factor. \(k\) denotes the number of rounds for ISP, looks forward. The ISPs do not know in advance the payoff of future rounds, they only have a priori knowledge.
EP\(_{(k)}\) denotes ISP\(_{i}\)'s belief about the expected access price in round \(k\), similarly \(E_{i}^{(k)}\) is the ISP\(_{i}\)'s expected subscriber number in that round. The payoff function of the service providers is

\[
\Pi_{i} = l_{i}^{*}p_{i} + \sum_{k=1}^{k} \Theta^{k}E_{i}^{(k)}p_{i}^{(k)}
\]

We assume that the ISPs have complete information when they select their strategies. However, real ISPs usually do not know the exact values of the parameters, e.g. the scope of other ISP’s payoff, the price of the Internet access in the next rounds, the number of own subscribers, and the discount factor of the ISPs can also be variable. These uncertainties have an impact on the price setting decisions what we quantify later on.

The formal definition of analysed long-term price setting game is as follows.

- **Players:**
  the Internet Service Providers, \(i=1,\ldots,n\), ISP\(_{i}\) is an incumbent with \(l_{i}\) loyal customers while ISP\(_{n+1}\) is the entrant of the game.

- **Strategies:**
  \(p_{i} \in [0, \alpha]\) the price of the Internet access at ISP\(_{i}\), ISP\(_{i}\) looks forward \(k_{i}\) rounds in order to maximize its payoff, only pure strategies are allowed.

- **Payoff functions:**
  the payoffs of the ISPs are
  \[
  \Pi_{i} = \left(1 - \frac{p_{i} - p_{n+1}}{\alpha}\right)l_{i}p_{i} + \sum_{k=1}^{k} \Theta^{k}\left(1 - \frac{p_{i} - p_{n+1}}{\alpha}\right)l_{i}p_{n+1}
  \]
  \[
  \Pi_{n+1} = \sum_{k=1}^{k} \frac{p_{i} - p_{n+1}}{\alpha}l_{i}p_{i} + \sum_{k=1}^{k} \Theta^{k}\frac{p_{i} - p_{n+1}}{\alpha}l_{i}p_{n+1}
  \]

The first term of the payoff function denotes the profit of the current round, while the second describes the weighted profit of future rounds. The profit is proportional to the number of subscribers and the profit of an access. The entrant ISP\(_{n+1}\) has to set the lowest price in order to have customers, otherwise ISP\(_{n+1}\) cannot enter the market. The algebraic calculation, based on the best response functions of the providers, yields implicit equations for the equilibrium prices. The equilibrium prices can be expressed explicitly solving a system of linear equations formulated from the implicit equations.

However, the solutions of the system are equilibrium prices only if all the prices are between 0 and \(\alpha\). In other cases, the optimal prices can be computed creating an order of the ISPs, e.g. based on their number of customers, the last ISP in the hierarchy is the entrant company. Using the ordering, the equilibrium prices can be expressed step-by-step using the backward induction paradigm of game theory. In particular, first the entrant ISP selects its equilibrium price in every possible scenarios which can exist based on the decisions of the other ISPs, after that the next ISP selects its price, etc. In the followings, we present simulation results where the equilibrium prices are computed using this backward induction method, moreover we also quantify the profits and market shares of the ISPs.

The impact of incumbent’s discount factor is shown when a new ISP enters a monopolistic local ISP market in Figure 4. The market share of the incumbent ISP is 100% (\(l_{1} = 100\)), she looks forward \(k_{1} = 5\) rounds. The access prices can be between 0 and 100 while the expected future price is 40. The entrant ISP has a discount factor of 0.8 and she looks forward 15 rounds while setting the price. At smaller discount values the possible future incomes are negligible to the income of the current round, therefore the incumbent ISP sets a price as high as possible (a), to maximize its profit. Accordingly, the entrant ISP can grab a significant part of the market with a small enough price, in some cases almost the half of the users select the entrant ISP (b). As the expected profit of the future becomes more significant, meaning the discount factor is more than 0.8, the incumbent ISP tries to hold its subscribers by lowering its price. Because of the lower prices the profit of the ISPs are decreasing (c), but the incumbent ISP has always larger profit than the entrant ISP.

We illustrate the impact of the scope of the pricing decisions on market shares. In Figure 5, we present three scenarios where the scope of incumbent ISP\(_{2}\) is 5, 10, and 20 rounds respectively, while the other ISPs look forward for 15 rounds. The incumbent ISPs weight the future incomes with a discount factor of 0.9 while the entrant has only 0.2 as a discount factor. If the number of rounds is small (a) ISP\(_{2}\) lost almost all of her custo-
mers, while if she sets prices based on 10 rounds her market share is the same as the entrant’s (b). Furthermore, if ISP2 prices Internet access based on long-term decisions, where she looks forward 20 rounds, the entrant ISP’s market share becomes marginal (c).

5. Conclusions

In this paper, we have demonstrated how Internet Service Providers can include loyal customer segments in their price setting strategies in order to maximize their incomes. We have provided game-theoretical analysis for handling disloyal customers on static ISP markets, where the number of providers is constant. It turned out that if disloyal subscribers exist in the market, the ISPs have to price Internet access using mixed strategies, creating competition between the ISPs. In addition, we have applied linear price different dependent loyalty model to price access on dynamic ISP markets.

Based on the simulation results, we state that long-term Internet access pricing strategies have to be selected carefully in order to maximise the profits of ISPs.

References